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SEMIANNUAL STATUS REPORT

OF

NASA RESEARCH GRANT NO. NsG-568 [redacted]

TO [redacted]

KENT STATE UNIVERSITY, KENT, OHIO [redacted]

Title of Project: Stochastic Models for Multi-valued, Multi-dimensional Relations

Principal Investigator: Triloki N. Bhargava

Period Covered: October 15, 1963, to April 15, 1964

UNPUBLISHED PRELIMINARY DATA

Date: April 16, 1964

Signature of Principal Investigator T. N. Bhargava

Title: Assistant Professor of Mathematics and Statistics

Address: Kent State University

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PRELIMINARIES

This is intended to be a semiannual status report of the NASA research grant number NsG-568 for support of the research project entitled "Stochastic Models for Multi-valued, Multi-dimensional Relations". While the award of the grant was conveyed to the principal investigator by Dr. Wilson, Chief of Mathematical Sciences Division, NASA, on October 10, 1963, the approximate beginning date of the project in the grant is designated as December 1, 1963. This report covers an official period of approximately six months from October 15, 1963, to April 15, 1964. At the same time, it must be noted that the research work on this project has been going on from about January, 1963, when the idea was first conceived by the principal investigator. Consequently, this report represents the sum progress made on this project from January, 1963; the financial statement for the first quarter (January 1, 1964, to March 31, 1964) is being submitted separately by the business manager of Kent State University.

For the sake of clarity, the report is divided into the following parts:

I. Background

II. Product, Future Direction of Research, Current Research Activities

III. Abstracts of Research Papers

IV. Appendices

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I. BACKGROUND AND CHRONOLOGICAL DEVELOPMENT

In December, 1963, the principal investigator joined the faculty at Kent State University, and in January, 1964, started working towards extensions and generalizations of some of the results obtained in his doctoral thesis entitled "Stochastic Model for a Binary Dyadic Relation, with Application to Group Dynamics". (Bhargava [1]). In spite of definite possibilities of further research (and important applications) using the approach put forward by the principal investigator (that is, studying time changes by means of changing directed graphs), the progress was rather slow mainly due to lack of time, research assistants, and other facilities. By March, 1963, the principal investigator had found certain interesting applications of his model in the fields of social and biological sciences. (Bhargava [2], Bhargava and Katz [3]). Furthermore, a graduate student, Mr. T. J. Ahlborn [for resume, see appendix IV.1], earning his way as an assistant instructor in the department, felt interested in the theory of directed graphs and started working on his master's thesis under the supervision of the principal investigator.

At this time it was felt that the research work would be greatly facilitated if some financial support could be found so as (i) to enable the principal investigator and other research personnel to devote more time to the project, and (ii) to find ways and means of arranging research meetings and discussions with other mathematicians working in the same or related fields. Consequently, a proposal was submitted to NASA in April, 1963, while the research work was still being carried out within the limited time and resources available to the principal investigator. During the summer of 1963 the principal investigator attended the Institute on "Inference in Stochastic

Processes" sponsored by the IMS and NSF at Michigan State University, supported by a research stipend under NSF grant number G-18976. At Michigan State University, Dr. S. D. Chatterji [for resume, see appendix IV.2] showed a definite interest in the project and it was agreed that Dr. Chatterji and the principal investigator should try to get together whenever possible to discuss the project further. Interest was also shown by Dr. J. Gani [for resume, see appendix IV.3] and Dr. P. H. Doyle [for resume, see appendix IV.4]. (Out of these three people Dr. Chatterji has already spent a week at Kent State University and contributed a great deal towqrds the progress of the project by his stimulating discussions and through his work on related topic of "counting topologies". Professor Gani and Professor Doyle have promised to be with us in the later part of this academic year.)

The award of the grant in October, 1963, proved to be of great help and quickened the pace of the progress of the research. Arrangements were made to obtain release of Mr. T. J. Ahlborn from his duties as an assistant instructor, and he was appointed as a research assistant (effective January, 1964). Mr. C. H. Curtis, another assistant instructor [for resume, see appendix IV.5] was hired as the other research assistant, and the department agreed to release him also from his duties (effective January 1, 1964). Both Mr. Ahlborn and Mr. Curtis are working on the project and have been supported by the project since then. Dr. S. D. Chatterji took a week off (from Mathematics Research Center, U.S. Army, University of Wisconsin) from February 17 to February 24, 1964, and spent the time at Kent State University working on the project. His contributionshave been very helpful, and he plans to continue working on the project. It is expected that a few other experts in the field of stochastic models and related fields will also visit us and spend some time on the project.

As a consequence of the above, the progress in the research has been rather satisfactory and some interesting and useful results have been obtained. These are described in the next part.

II. PRODUCT, CURRENT RESEARCH ACTIVITIES, AND FUTURE DIRECTION OF RESEARCH

The following may be described as some of the important results obtained so far:

A. A Link-up Between Directed Graphs (or simply, Digraphs) and Point-set Topology. Two research papers on this interesting tie-up between the theory of digraphs and point-set topology have been prepared - one of which was presented (by title) at the joint annual meetings of the AMS and MAA at Miami, Florida, by the principal investigator (T. N. Bhargava) in January, 1964, and the other is to be presented at the regional meetings of AMS at New York City jointly by the principal investigator (T. N. Bhargava) and one of the research assistants (T. J. Ahlborn) in April, 1964. It is believed that this contribution is of far-reaching importance, opening an entirely new way of looking at digraphs by associating topological spaces with the digraphs, and as a consequence making it possible to use the methods and mathematics of general topology in the theory of graphs and digraphs. Abstracts of these two papers are given in the parts III.1 and III.2. It is proposed to write the results of the above two papers in one joint paper (by the principal investigator, T. N. Bhargava, and Mr. T. J. Ahlborn), and submit it for publication to Mathematische Annalen by June, 1964.

B. Counting Topologies. To be able to exploit fully the topological spaces associated with digraphs, it is necessary to find methods for counting topologies which in turn will enable us to describe probability distributions on the appropriate spaces. Dr. S. D. Chatterji has been able to develop such counting methods and his results are presented in form of an abstract in part III.3. Dr. Chatterji expects to have it ready for publication purposes by September, 1964.

C. <u>Most General Way of Describing a Graph and a Digraph.</u> In developing stochastic models for time changes in a multi-valued, multi-dimensional relation, the first major problem that one encounters is that of defining the most general kind of digraph. This necessitates extension of the notion of the simplest kind of digraph which consists of no loops and at most one edge between any ordered pair of points. This is accomplished by defining a digraph as an ordered pair with the set A as the first element and a subset E_P of the product set AxA as the second element, where E_P is defined as the union of subsets E_i obtained by means of a partition P of the product set AxA, such that $E_i \cap E_j$ is empty for every $i \neq j$, and $E_P = \bigcup_{i=i}^{k} E_i$. Proper choice of partition P gives rise to different kinds of digraphs, and a suitable function f defined from an Index set I to itself yields different notions of connectivity. In general, given a partition P and a function f we get what may be called as $E_{P,f}$ - connectedness. This idea is being developed further to obtain generalized digraphs with various types of connectedness, including a generalization of the connectivity and accessibility classification developed by the principal investigator in his earlier work [1].

D. Random Graphs and Digraphs. Various interesting possibilities present themselves when one tries to study the evolution of a random graph as first studied by Erdos and Renyi [4]. We (the principal investigator and Dr. S. D. Chatterji) are investigating a more general and possibly more fruitful approach to the same. It consists in defining a random evolutionary graph as follows: one first picks a point at random from a collection on N points and with a certain probability one moves from that point to another. This obviously defines a directed edge. From the point arrived at, one then moves to another point according to the probability distribution which depends on the previous points visited. The possibility of repeating points that have been visited before or visiting itself is not excluded. There are two simple cases that one should study. First, the case where the points are chosen in a Markov sequence. One should notice that this scheme is not quite the same as the schemes studied by Erdos and Renyi or in the other two papers referred to by Erdos and Renyi. A more general way of defining a random evolutionary graph which would include not only the one just described but also the ones contained in the previously mentioned papers seems to be as follows: one first chooses now a random subset from the set of all vertices, and then one chooses another random subset which may be quite arbitrary and then chooses a random mapping from the first chosen subset to the second. Next, one might choose a third random subset; the probability distribution according to which this third subset is chosen may depend on the first two choices. A random mapping is again chosen between the second and the third subset, and then one might proceed to choose a fourth random subset and another random mapping between the third and fourth. Each random mapping defines a class of edges, and so at any particular stage one has a graph consisting of randomly chosen edges. One can study such things as the nature of the graph after a certain number of stages; how many components it has; whether or not it is connected; what or how many cycles there are; and what sort of trees it may or may not have. This formulation clearly contains all the previous formulations referred to above.

E. Information Theory and Theory of Graphs (Digraphs). This part of the research project is concerned with an investigation of the different link-ups between various purely mathematical theories through the usage of the information concept, and of the important applications of the information-theoretic notions to the fields of probability, communication theory, and the theory of graphs. Although the concept of information is a relatively new concept in the domain of mathematics, its use and applications in mathematics itself and areas depending on the probabilistic theories in mathematics have been manifold and far-reaching. Specifically the purpose of the proposed research is to develop a suitable notion of entropy of certain kinds of stochastic processes (which describe time-changes in a binary dyadic relation over a finite set of points; or equivalently time-changes in a directed graph or an incidence matrix) and its connection with a notion of the entropy in ergodic theory. It is being carried out jointly with Dr. S. D. Chatterji.

F. <u>Historical Survey of the Theory of Digraphs.</u> There does not exist in literature any uniform treatment of the theory of digraphs, particularly with a viewpoint of its application to probability and statistics. Mrs. Sigrid Ohm and Miss Mildred C. Wilson [for resumè, see appendix IV.6 and IV.7], two graduate assistants in the department, are trying to collect all the relevant material in this field and present it in a uniform manner under the supervision of the principal investigator. Neither of them is supported by NASA at present.

G. <u>Digraphs and Weak Metric.</u>

<u>Problem:</u> To investigate the relation between the incidence matrix of a digraph $\Gamma(A)$ on set A of points and possible weak metrics (a weak metric is a real-valued function $d(a,b)$ defined over the set of all pairs of elements of a set A such that (i) if $a = b$ then $d(a,b) = 0$, and (ii) for all a, b, c in A, $d(a,b) + d(b,c) = d(a,c)$) which may be defined on the directed graph.

Investigation of this problem has just begun, and Mr. T. J. Ahlborn is working further on the same. Up to this time we have considered directed graphs on only a finite set of points. It appears as if the concept of a weak metric will enable us to study directed graphs containing any number (finite or infinite) of points. This would expand and deepen our understanding of the directed graph as a mathematical model.

III. Abstracts of Research Papers

AND

IV. Appendices (Resume)

III.1 SOME RESULTS IN THEORY OF DIGRAPHS

T. N. Bhargava

Let $\Gamma(A)$ be a digraph (directed graph) defined on the set A consisting of N points, and let $S = \{\Gamma(a), a \subset A\}$ be the set consisting of all subdigraphs, on n points, of the digraph $\Gamma(A)$. A partition $\rho = (\pi_1, \pi_2, ----, \pi_p)$ defined on S is such that each member $\Gamma(a)$ of S belongs to one and only one of the classes $\pi_1, \pi_2, ----, \pi_p$, and none of these classes is empty. In this paper, partitions based on various notions of accessibility (a point j is said to be accessible from another point i if there exists a directed path from i to j) are presented; identification and counting theorems are derived in terms of incidence matrix $C(A)$ which is an equivalent representation of $\Gamma(A)$ in terms of a matrix. A topology $\mathcal{T}$ is established on $\Gamma(A)$ by defining $\Gamma(a)$ to be open if none of the points in the subset a is accessible from any of the points in the subset $A-a$; it is shown that $\mathcal{T}$ has many well-known properties; for example it is a discrete space of Aleksandrov. Some interesting results are obtained in terms of the partitions π and the topology $\mathcal{T}$ by introducing the idea of the 'core' of a point (core of a point i is defined to be the intersection of the smallest open and closed sets containing i), and the 'core function'. Certain statistical results for digraphs are obtained by introducing probability measures on S. Finally a discussion of the general case of graphs with colored edges is also presented, and some of the results for the digraphs are extended for the general case.

III.2 DIRECTED GRAPHS AND POINT-SET TOPOLOGY

T. N. Bhargava and T. J. Ahlborn

Let A be a finite set consisting of N points and let $\mathcal{E} = \{E\}$

be the family of all subsets E of the cartesian product $A \times A$.
The pair (A,E) has three isomorphic representations in terms of:
(i) an aggregate of binary dyadic relations on the set A; (ii) a directed graph (digraph) $\Gamma(A)$, and (iii) an incidence matrix $C(A)$.
By suitably defining an open or a closed set, every $E \in \mathcal{E}$ determines a topology $(A,\mathcal{T}_E)$ on A, where $\mathcal{T}_E = \{B \mid B \subseteq A$ and B is open with respect to $E\}$. We find that the topology $(A,\mathcal{T}_E)$ determined by E on set A has complete additive closure. Let a set $B \subseteq A$ be open with respect to E if for all $i \in (A-B)$, $j \in B$, we have $\langle i,j\rangle \in (A \times A - E)$.
Various properties of such topologies are obtained, for example: $(A,\mathcal{T}_E)$ is T_1 if and only if $E = \Phi$. Furthermore, if $U = \{(A,E) \mid E \in \mathcal{E}\}$ and if $S = \{(A, E) \mid$ if $\langle i,j\rangle \in E$ and $\langle j,k\rangle \in E$, then $\langle i,k\rangle \in E\}$, (a digraph belonging to S is called a transitive digraph), then the mapping $S \rightarrow U, \left((A, E) \rightarrow (A, \mathcal{T}_E)\right)$, is a one-to-one mapping of S onto U. Finally some properties of a topology induced on a partitioning of the points of set A by means of the "core function" (see abstract number 64T-138, AMS Notices, February, 1964) are investigated.

III.3 COUNTING THE NUMBER OF TOPOLOGIES ON n POINTS

S. D. Chatterji

Let $\Omega_n = 1, 2, 3, \text{---} n$. A topology on Ω_n is a collection $\mathcal{T}$ of subsets of Ω_n such that

(1) $\Phi, \Omega_n \in \mathcal{T}$

(2) $A, B \in \mathcal{T} \Rightarrow A \cap B \in \mathcal{T}, A \cup B \in \mathcal{T}$

Let $t(n)$ = the number of distinct topologies on Ω_n.

We have tried to solve the problem of obtaining an analytic expression for $t(n)$. The problem seems quite hard, and we have only been able to obtain certain inequalities for $t(n)$. We have also noticed a connexion with counting the numbers of graphs of a certain type.

It can be shown that a topology on Ω_n can be specified equally well by a mapping C from Ω_n to $P(\Omega_n)$ - the power set or set of subsets of Ω_n. The mapping C must have the following properties:

(1) For all $i \in \Omega_n$, $i \in C(i)$

(2) If $j \in C(i)$, then $C(j) \subset C(i)$

$C(i)$ can be thought of as being the closure of the point "i". That such a function can be used to establish a topology on Ω_n may be verified by using the Kuratowski closure postulates. The problem then is to count the number of such mappings. Clearly $C(i)$ can be chosen in 2^{n-1} ways. So $t(n) < 2^{n(n-1)}$. On the other hand, the number of algebras of subsets on Ω_n (call it $a(n)$) is clearly less than or equal to $t(n)$. Asymptotic expressions for $a(n)$ are known. Actually, $a(n)$ can be written down explicitly using number-theoretic partition-functions. Thus we have:

<u>Theorem</u>: $a(n) \leq 2^{n(n-1)}$

(Note: The r.h.s. of the inequality is strict except for $n = 1$ and 2. The l.h.s. is strict for $n > 1$.)

It is known that (see Bhargava and Ahlborn [III.2]) one can set up a one-to-one correspondence between the topologies on Ω_n and the number of accessibility digraphs on n-vertices. (An accessibility digraph is one in which either two vertices are joined by a directed edge or else there is no string of properly directed edges connecting them.)

Many other problems of this type of counting can be posed. For example, how many topologies are T_0, (there is only one T_1 topology), how many connected, how many non-homeomorphic, etc. Further, what proportion of topologies in $\Omega_m \times \Omega_n$ are product topologies? We propose to go into these matters later.

IV.1 RESUME

Name: Thomas J. Ahlborn

Date and Place of Birth: October 12, 1940 Charleroi, Pennsylvania

Marital Status: Single

Present Position: Research Assistant, NASA Project NSG-568

Educational Experiences:

Degrees:

High School	Rostraver High School, Pricedale, Pennsylvania	1958
B. S. in Education	California State College, California, Pennsylvania	1962
M. S. (Mathematics)	Kent State University, Kent, Ohio	1964

Teaching and Training:

Charleroi High School, Pricedale, Pennsylvania	Teach eighth grade mathematics	1962 (summer)
Kent State University, Kent, Ohio	Graduate Assistant Intermediate Algebra	1962-1963
Charleroi High School, Pricedale, Pennsylvania	Teach Algebra I	1963 (summer)
Kent State University, Kent, Ohio	Assistant Instructor Beginning Algebra, Arithmetic Review	1963 (fall)
Kent State University, Kent, Ohio	Research Assistant Graph Theory	1964 (winter, spring)
University of Rochester	Teaching Assistantship	1964 (fall)

Remarks: Member of American Mathematical Society, Mathematical Association of America, NSF Summer Fellowship for Graduate Teaching Assistants for summer of 1964.

IV.2 RESUME

Name: Srishti D. Chatterji

Date and Place of Birth: June 29, 1935 Calcutta, India

Marital Status: Single

Present Position: Research Mathematician
Mathematics Research Center
University of Wisconsin

Educational Experiences:

Degrees:

B. S.	Lucknow University, India Mathematics, Statistics	1952
M. S.	Lucknow University, India Mathematical Statistics	1954
Ph. D.	Michigan State University Statistics, Mathematics	1960

Teaching and Training:

Indian Statistical Institute, India	Research	1955-1956
Syracuse University	Graduate Assistant	1956-1958
Michigan State University	Graduate Assistant	1959-1960
University of New South Wales, Australia	Teaching and Research	1960-1962
Michigan State University	Teaching and Research	1962-1963
Mathematics Research Center, University of Wisconsin	Research	1963-

Field of Present Major Scientific Interests:

Mathematical Statistics, Applied Probability, Probability, Mathematics

IV.3 RESUME

Name: Joe Gani

Professor Gani received his bachelor's degree from the University of London and his D. I. C. from the Imperial College, London. His doctorate work was done at the Australian National University, Canberra. He has taught at the University of Melbourne, Western Australia University, Australian National University, London University and in this country at Columbia University, Stanford University, and John Hopkins University. Presently he is professor of statistics at Michigan State University. Professor Gani is a specialist in the fields of probability and statistics. He is internationally known, particularly in his work on probabilistic models. He has written numerous papers in his field which have been published in the journals of international repute. His present interest is in stochastic processes and biological models.

IV.4 RESUME

Name: Patrick H. Doyle

Professor Doyle received his bachelor's and master's degrees from the University of Michigan and his Ph. D. from the University of Tennessee. He has taught at Western Michigan University, the University of Tennessee, and Michigan State University and is currently professor of mathematics at Virginia Polytechnic Institute. Professor Doyle is a specialist in the field of topology and has published numerous papers in the field.

IV.5 RESUME

Name: Clifford H. Curtis

Date and Place of Birth: February 23, 1939 Cleveland, Ohio

Marital Status: Married

Present Position: Research Assistant

Educational Experience:	Case Institute of Technology	2 years
	Kent State University	4 years

Degrees:

B. S. in Education	Kent State University, Kent, Ohio

Teaching and Training:

Kent State University, Kent, Ohio	Graduate Assistant

Remarks: Member of Pi Mu Epsilon, Mathematical Association of America

IV.6 RESUME

Name: Sigrid Ohm

Date and Place of Birth: December 19, 1942

Marital Status: Married

Present Position: Assistant instructor

Educational Experience:

Degrees:

B. S. in Education	Kent State University, Kent, Ohio	1963
M. A. (working on)	Kent State University, Kent, Ohio	

Teaching and Training:

Kent State University, Kent, Ohio	Assistant instructor	1963-1964

Fields of Present Major Scientific Interests:

Graph theory, Algebra

Remarks: Member of Pi Mu Epsilon, Kappa Delta Pi, graduated summa cum laude, NSF summer and cooperative fellowship.

IV.7 RESUME

Name: Mildred C. Wilson

Date and Place of Birth: August 3, 1941 Charlotte, North Carolina

Marital Status: Single

Present Position: Graduate Assistant

Educational Experiences:

Degrees:

High School (Honor graduate)	Walter Bickett High School Monroe, North Carolina	1959
B. S. in Mathematics (Cum Laude) Teacher Certification	Presbyterian College Clinton, South Carolina	1963
M. A. (working on)	Kent State University Kent, Ohio	

Teaching and Training:

Presbyterian College Clinton, South Carolina	Mathematics tutor, in charge of college mathematics laboratory, College Algebra, Trigonometry, Analytic Geometry	1961-1963
Clinton High School Clinton, South Carolina	Substitute Teaching, Algebra I and II, Geometry, Trigonometry, Advanced Algebra	1962-1963
Kent State University Kent, Ohio	Graduate Assistant Teaching, Remedial Freshman Mathematics	1963-

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